## CHAPTER 4

## RATIO, PROPORTION AND UNITARY METHOD

## **INTRODUCTION**

## RATIO

See, Rohan and Sonal have collected some stamps.



How will you compare the number of stamps collected by Rohan and Sonal?

There are two ways of comparing.

#### (i) By finding difference.

Rohan has 5 stamps more than Sonal.

$$15 - 10 = 5$$
  $\blacksquare$  We have compared by taking the difference.

#### (ii) By dividing numbers.

Rohan's collection  $\rightarrow$   $\frac{15}{10} = \frac{3}{2}$  (lowest form)

Rohan's collection is  $\frac{3}{2}$  times the collection of Sonal.

When we compare two quantities of same kind by division, we say a ratio is formed.

Comparing two quantities by division is called **Ratio**.

We use the symbol ':' to denote ratio.

The ratio form of  $\frac{3}{2}$  is **3** : **2** and is read as 'three **is to** two'.

In the ratio 3 : 2, 3 and 2 are called the terms of the ratio.

#### 3 is called the First term or Antecedent.

2 is called the **Second term** or **Consequent**.

Fractional 
$$\frac{3}{2}$$
 First term  $3:2$  Ratio form   
form

Now look at the table.

Fraction	Ratio	Ratio in simplest form	Read as
<u>14</u> 16	14 : 16	7:8	7 is to 8
<u>100</u> 40	100 : 40	5 : 2	5 is to 2
<u>19</u> 25	19 : 25	19 : 25	19 is to 25
$\frac{8}{40}$	8 : 40	1 : 5	1 is to 5

#### Remember

While forming a ratio-

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- The two quantities compared should be expressed in the same unit. e.g. ratio of 2 m to 75 cm should be 200 cm to 75 cm.
- The names/units of the two quantities should not be used in ratio. e.g. ratio of 3 m to 2 m is 3 : 2 not 3 m : 2 m.
- The two quantities compared should be of the same kind.
   e.g. ₹ 20 and 5 apples cannot be compared.
- Ratio should be always expressed in the simplest form e.g. 17 : 51 should be written as 1 : 3.
- The order of the terms in a ratio is important.

e.g. ratio of boys to girls in a class is 2 : 3 while the ratio of girls to boys is 3 : 2.

#### Note:

A ratio gives the relative size of two quantities and not the actual amount of the two quantities.

e.g. the ratio of red balls to yellow balls in a box is 3 : 4. This ratio tells us that for every 3 red balls there are 4 yellow balls. It does not tell us that the number of red balls is equal to 3 and yellow balls is equal to 4.

#### Let us understand the following examples.

**Example 1:** Express the following pairs into ratio in the simplest form.

(a) 4 m long cloth

2 m long cloth

Ratio of the lengths of cloth

$$= 4 : 2$$
  
 $= \frac{4}{2} = \frac{2}{1}$   
 $= 2 : 1$ 

(b) 150 kg

120 kg

Ratio of the weight of sugar in the bags

$$= 150 : 120$$
$$= \frac{150}{120} = \frac{5}{4}$$
$$= 5 : 4$$

(c) 12 l

9 I

Ratio of the quantity of milk

$$= 12:9$$
$$= \frac{12}{9} = \frac{4}{3}$$
$$= 4:3$$



**Example 2:** Find the ratio of 3 kg to 250 gm.

**Solution:** We know 3 kg = 3000 gm

\_\_\_\_\_3 × 1000

So, Ratio of 3 kg to 250 gm = Ratio of 3000 gm to 250 gm

 $= \frac{3000}{250} = \frac{12}{1}$ = 12 : 1

- Example 3: Mr Gupta earns ₹ 4,500 in a month. He spends ₹ 3,600 and saves the rest. Find the ratio of—
  - (a) earnings to expenditure.
  - (b) expenditure to savings.
  - (c) savings to earnings.

Solution: Mr Gupta's earnings = ₹4,500

Expenditure = -₹3,600

∴ Savings = ₹ 900

(a) Ratio of earnings to expenditure = 4500 : 3600

$$=\frac{54500}{43600}=\frac{5}{4}$$
 or 5:4

(b) Ratio of expenditure to savings = 3600 : 900

$$=\frac{43600}{1900}=\frac{4}{1}$$
 or  $4:1$ 

(c) Ratio of savings of earnings = 900 : 4500

$$=\frac{1900}{54500}=\frac{1}{5}$$
 or 1:5

So, the required ratios are 5 : 4, 4 : 1 and 1 : 5.

#### Worksheet 1

1. Express the following as ratios.

(a)	$\frac{3}{5}$	(b)	<u>13</u> 17
(c)	<u>49</u> 36	(d)	1000 537
(e)	<u>91</u> 92	(f)	<u>1</u> 49
(g)	15	(h)	<u>42</u> 13

2. Find the ratio of the shaded region to that of the whole figure.



#### 3. Express the following ratios in the simplest form.

- (a) 36 m to 16 m (b) ₹ 125 to ₹ 35
- (c) 324 : 144 (d) 65 books to 91 books
- (e) 125 : 1125 (f) 45 hours to 36 hours

#### 4. Express the following as ratio in simplest form.

- (a) 10 cm to 10 m
  (b) 45 minutes to 4 hours
  (c) 35 days to 3 weeks
  (d) 2 dozen to 1 score
- (e) ₹ 3.50 to 75 paise (f) 4 kg to 250 gm
- 5. Which of the following pairs of ratio is greater?
  - (a) 5:8 or 4:5 (b) 9:11 or 11:9
  - (c) 23:24 or 15:16 (d) 3:10 or 15:40
- 6. There are 50 students in a class. If 24 of them are boys, find the ratio of boys to girls.

- 7. Sahil ran a distance of 1.5 km and his younger sister could run only 500 m. Express the distances as a ratio.
- 8. Out of 32 metre long cloth, 24 metre was used for making eight frocks. Find the ratio of-
  - (a) total cloth and cloth used. (b) remaining cloth and cloth used.
- 9. Mr Arun earns ₹ 9,500 per month and his wife earns ₹ 8,000. Find the ratio of—
  - (a) Mr Arun's income to his wife's income. (b) wife's income to total income.
- 10. In a dictation test of 20 words, Rohan spelled 18 words correctly. Find the ratio of—
  - (a) total words to wrongly spelled words.
  - (b) correctly spelled words to wrongly spelled words.
- 11. Mrs Sareen earns ₹ 2,50,000 every year and pays ₹ 24,000 as income tax. Find the ratio of—
  - (a) income tax to income. (b) income to income tax.
- 12. Fill in the following blanks.

(a) Comparing two quantities by division is called \_\_\_\_\_.

- (b) A ratio is always expressed in its \_\_\_\_\_ form.
- (c) The first term of a ratio 11 : 24 is \_\_\_\_\_ and the second term is \_\_\_\_\_.
- (d) The ratio of the letter 'M' in the word MATHEMATICS to the total letters in the word is \_\_\_\_\_.
- (e) The ratio of even numbers to odd number in a set of natural numbers from 1 to 25 is \_\_\_\_\_.

## PROPORTION



See, Neha has purchased 6 pens at ₹ 8 each. Cost of 6 pens =  $6 \times \overline{<} 8$ =  $\overline{<} 48$ Ratio of the number of pens = 6:10= 3:5Same Cost of 10 pens =  $10 \times \overline{<} 8$   $= \overline{<} 80$ Ratio of the cost of the pens = 48:80= 3:5Same

We have, 6:10 = 48:80

Such an equality of ratios is called **proportion**.

A statement of equality of ratios is called **Proportion.** 

We say the four numbers 6, 10, 48 and 80 are in proportion.

The symbol used to denote proportion is (: :).

We write 6 : 10 : : 48 : 80.

We read 6 is to 10 **as** 48 is to 80.

The numbers 6, 10, 48, 80 are called the **terms** of the proportion.

We have,



Now, let us take the product of means and extremes.

Product of means  $= 10 \times 48 = 480$ Product of extremes  $= 6 \times 80 = 480$ 

If four numbers (quantities) are in proportion, then the product of the extremes is equal to the product of the means.

#### Note:

The above property helps us in checking whether the given four terms are in proportion.

Now let us take some examples.



**Example 6:** Are 20, 10, 10, 5 in proportion?

Solution: Product of extremes =  $20 \times 5 = 100$ Product of means =  $10 \times 10 = 100$ So, 20, 10, 10, 5 are in proportion.

#### Note:

In this example, the second term and the third term are repeated. We say that 20, (10), 5 are in continued proportion. The number 10 is written only once.

**Example 7:** Are 16, 8 and 4 in continued proportion?

Solution: Here, we have to find out if 16, 8, 8, 4 are in proportion. (Why?)

Product of extremes =  $16 \times 4 = 64$ 

Product of means =  $8 \times 8 = 64$ 

So, 16, 8, and 4 are in continued proportion.

#### Worksheet 2

1.	Are the following numbers in propor	tion?
	(a) 9, 12, 18, 24	(b) 3, 4, 8, 16
	(c) 22, 33, 16, 24	(d) 63, 55, 32, 72
	(e) 4, 9, 32, 72	(f) 15, 75, 45, 120
2.	Fill in the following boxes so that the	e four numbers are in proportion.
	(a) 32, , 6, 12	(b) 21, 27, 14,
	(c) 15,, 27, 36	(d) 33, 132,, 16
3.	Are the following in continued propo	ortion?
	(a) 27, 36, 42	(b) 3, 9, 27
	(c) 48, 36, 27	(d) 36, 90, 75
4.	Fill in the boxes so that the three nu	mbers are in continued proportion.
	(a) 6, 18,	(b) 25, 20,
	(c) , 32, 64	(d), 60, 45

#### 5. Determine if the following ratios are in proportion.

- (a) 48 kg : 6 kg and 25 g : 200 g
- (b) 8 m : 21 m and ₹ 24 : ₹ 63
- (c) 45 girls : 60 girls and 48 boys : 64 boys
- (d) 5.21:3.9 1 and 3 ml: 4 ml

#### 6. Fill in the following blanks.

- (a) Equality of two ratios is called \_\_\_\_\_.
- (b) A proportion has \_\_\_\_\_ terms.
- (c) The first and fourth terms of a proportion are called \_\_\_\_\_\_.
- (d) The \_\_\_\_\_ and \_\_\_\_\_ terms of a proportion are called means.
- (e) For four numbers to be in proportion, the product of \_\_\_\_\_ should be equal to product of \_\_\_\_\_.

#### **UNITARY METHOD**

#### Let us consider a situation.

It is Rohan's birthday party. He bought 15 geometry boxes for ₹ 450 to be given as return gifts. But, four more friends came for the party.

Let us find how much more money he needs to spend on four more geometry boxes.



So, the cost of four more geometry boxes is ₹ 120.

The method of first finding the value of one article/unit by division and then, finding the value of more articles/units by multiplication is called **Unitary Method**.

#### See one more example.

**Example 8:** A car travels a distance of 550 km in 5 hours.

- (a) What distance will the car cover in 7 hours?
- (b) How many hours are needed to travel a distance of 3080 km if the car travels at a uniform speed?

#### Solution: We have,

(a) Distance travelled by the car in 5 hours = 550 km Distance travelled by the car in 1 hour = 550 km ÷ 5 = 110 km Distance travelled in 7 hours = 110 km × 7 = 770 km (b) Time taken to travel 550 km = 5 hrs Time taken to travel 1 km =  $\frac{5}{550}$  hrs =  $\frac{1}{110}$  hrs =  $\frac{1}{110}$  hrs Time taken to travel 3080 km =  $\frac{3080 \times 1}{110}$ = 28 hrs

#### Worksheet 3

- 1. Sahil buys eight books for ₹ 72. What is the cost 45 books?
- 2. The bus fare for 20 persons from Delhi to Chandigarh is ₹ 9,500. Find the bus fare for 19 persons.
- 3. Arun earns ₹ 3,90,000 per year. Find his earnings for 25 weeks.
- 4. The cost of 18 kg wheat is ₹ 216. Find the cost of one quintal wheat.
- 5. Four erasers are purchased for ₹ 10. Find the cost of one dozen erasers.
- 6. A worker earns ₹ 18,000 in 15 months.
  - (a) How much will the worker earn in seven months?
  - (b) In how many months will he earn ₹ 3,600?
- 7. The weight of 72 books is 9 kg.
  - (a) Find the weight of 80 such books.
  - (b) How many books will weigh 6 kg?

- 8. A factory produced 57900 screws in the month of April, 2015. (The factory produces same number of screws every day.)
  - (a) How many screws did the factory produce in 8 days?
  - (b) In how many days, did the factory produce 34740 screws?

## ALUE BASED QUESTIONS

Ramesh went to the market to buy potatoes. He found a shopkeeper selling
 2 1/2 kg potatoes for ₹ 30. Ramesh bought 15 kg potatoes from him and gave the shopkeeper ₹ 500 note. Shopkeeper gave back a balance of ₹ 350 to Ramesh.

On way back home he realised that the shopkeeper had returned him more money. Ramesh went back to the shop and returned the extra amount.

- (a) What was the cost of 15 kg potatoes?
- (b) How much money did Ramesh return to the shopkeeper?
- (c) What value is exhibited by Ramesh's act?
- 2. Mehak, Harpreet and Raj planted trees in their school and its neighbourhood during the *Vanamahotsav* Celebration. They were really very happy and excited to be a part of the celebrations. Mehak planted 18 trees, Harpreet planted 24 trees and Raj planted 32 trees.
  - (a) What is the ratio of trees planted by Mehak to those planted by Harpreet?
  - (b) How many more trees should Raj plant so that the ratio of trees planted by Mehak to that of Raj becomes 1 : 2?
  - (c) How do you celebrate *Vanamahotsav* in your school? Why should we take care of our environment?

### BRAIN TEASERS

- 1. A. Tick ( $\checkmark$ ) the correct answer.
  - (a) The ratio of 1 cm to 1 mm is-
    - (i) 1 : 10 (ii) 10 : 1 (iii) 1 : 100 (iv) 100 : 1

(b) Fill in the box so that , 1, 0.5 are in continued proportion. (i) 2 (ii) 0.4 (iii) 0.2 (iv) 4 (c) If 8 : 12 : : 26 : , then the number in the box is-(i) 39 (ii) 12 (iii) 16 (iv) 18 (d) The cost of six chocolates is ₹ 210, then the cost of four chocolates is-(i) ₹ 90 (ii) ₹ 120 (iii) ₹ 140 (iv) ₹ 100 (e) The ratio of a dozen to a score is-(i) 5:3 (ii) 2:3 (iii) 3:2 (iv) 3:5 B. Answer the following questions. (a) In the word 'GEOMETRY', find the ratio of vowels to the consonants. (b) 12 bags of wheat weigh 96 kg. How much will 20 similar bags weigh? (c) Which is greater—3:4 or 5:6? (d) Find the ratio of 125 ml to 5 litres. (e) Find the number in the box if (: 6 : : 55 : 11. Express the following ratios in the simplest form. 2. (a) 10 | to 0.25 | (b) 2 hours to 45 seconds 13 weeks to 1 year (d) 1 century to 1 decade (c) 3. Are the following in proportion? (a) 16, 64, 18, 72 (b) 1.8, 4, 0.6, 5 Fill in the boxes so that the numbers are in proportion: 4. (a) 24, 18, , 96 1000, 500, 200, (b) Sahil weighs 36 kg. His brother weighs 2 kg more. Find the ratio of the weight of 5. Sahil's brother to Sahil. 6. In an orchard, there are 150 trees. 65 of them are apple trees, 35 are pear trees and the rest are orange trees. Find the ratio of-

- (a) apple trees to pear trees.
- (b) orange trees to apple trees.
- (c) pear trees to total trees.

- 7. Cost of one dozen oranges is  $\gtrless$  45.
  - (a) Find the cost of one score oranges.
  - (b) How many oranges can be purchased for ₹ 60?
- 8. Cost of five pizzas is ₹ 725 and six pastries is ₹ 210. If I want to purchase three pizzas and 12 pastries, how much should I pay?
- 9. What is the ratio of the-
  - (a) number of even numbers to odd numbers from the set of natural numbers from 30 to 50?
  - (b) prime numbers and composite numbers from the set of natural numbers from 1 to 20?
- 10. The sides of two squares are in the ratio 5 : 7. Find the ratio of their areas.
- 11. A distance of 500 km between two cities Raipur and Rajmundri is represented on the map by 5 cm. What is the actual distance between the cities if they are 8 cm apart on the map?

## HOTS

The ratio of the length of a vertical pole and its shadow on the ground is 7 : 2. Find the length of the pole if the length of the shadow is 2.4 m.

## YOU MUST KNOW

- 1. Comparing two quantities by division is called ratio.
- 2. Ratio is always expressed in the simplest form.
- 3. The order of the terms is important in ratio.
- 4. When two ratios are equal. They are said to be in proportion.
- 5. If four numbers (quantities) are in proportion, then the product of the extremes is equal to the product of the means.
- 6. If three numbers (quantities) are in continued proportion, then square of middle number (quantity) is equal to product of first and third numbers (quantities).
- 7. The method of first finding the value of one article by division and then finding the value of more articles by multiplication is called Unitary Method.
- 8. The order of operations, i.e. division followed by multiplication is important.

## PERCENTAGE AND ITS APPLICATIONS

## **INTRODUCTION**

Do you remember percentage?

A fraction with denominator equal to hundred is called **Per cent**. e.g.  $\frac{19}{100} = 19\%$ ,  $\frac{2}{100} = 2\%$ , or  $\frac{5}{100} = 5\%$ , The symbol used for per cent is (%) which stands for **divided by 100**.

#### Let us take some problems.

1. Express the following per cents as fractions in lowest terms.

(a)	5%	>	$5\% = \frac{5}{100}$
(b)	$6\frac{2}{5}\%$		$=\frac{1}{20}$
(c)	40%		

- (d)  $33\frac{1}{3}\%$
- 2. Express the following per cents as decimals.



3. Write the following fractions or mixed numbers as per cents.



- 7. Ritu scored 68% in her Mathematics test. If maximum marks were 50, how many marks did she score?
- 8. I purchased 5 I milk and used 80% of it for making sweets. How much milk is left?

#### EXPRESSING ONE QUANTITY AS A PER CENT OF ANOTHER QUANTITY

Example 1: What per cent is ₹ 50 of ₹ 250?

Solution: Out of ₹ 250, ₹ 50 is written as  $\frac{50}{250}$ 

Now, converting into percentage

We have,

$$\left(\frac{150}{250}\times^{20}100\right)\% = 20\%$$

₹ 50 is 20% of ₹ 250.

**Example 2:** Out of 15 eggs, 3 eggs were broken. Find the per cent of broken eggs.

Solution: Number of eggs = 15

Number of broken eggs = 3

Percentage of broken eggs = 
$$\left(\frac{13}{15} \times {}^{20}100\right)\% = 20\%$$

20% of the eggs were broken.

#### Worksheet 1

- 1. What per cent is-
  - (a) ₹ 63 of ₹ 90 (b) 80 | of 240 |
  - (c) 16 of 250 (d) 200 cm of one metre
  - (e) 13 weeks of one year (f) one dozen of one score
- 2. There are 200 trees in an orchard. If 125 of them are apple trees, find the percentage of apple trees.
- 3. A man earns ₹ 12,000 and spends ₹ 10,500. If he saves rest of the money, find the percentage of his savings.
- 4. There are 700 students in a school out of which 420 are girls. Find the percentage of boys in the school.
- 5. Mansi obtained 410 marks out of 500 in her half-yearly examination while Neha, her friend got 534 marks out of 600 marks. Find whose performance is better.
- 6. What is the percentage of prime numbers from 1 to 10?

#### MORE ABOUT PERCENTAGE

Example 3: Find the amount 20% less than ₹ 150.

**Solution: Step 1:** Take 20% of ₹ 150.

20% of ₹ 150 = 
$$\frac{20}{100} \times 150$$
  
= ₹ 30

Step 2: Subtract 20% of ₹ 150 from ₹ 150.

So, ₹ 120 is 20% less than ₹ 150.

**Example 4:** Find the amount 5% more than 600.

Solution: Step 1: Take 5% of 600.

5% of 600 = 
$$\frac{5}{100} \times 600$$
  
= 30

Step 2: Add 5% of 600 to 600.

$$600 + 30 = 630$$

Therefore, 630 is 5% more than 600.

#### Worksheet 2

1. Find—

- (a) the amount 6% more than  $\gtrless$  120.
- (b) the amount 15% less than  $\gtrless$  400.
- 2. The monthly salary of Mohan is ₹ 950. If his salary increases by 6%, find his new salary.
- 3. A dealer buys an old car for ₹ 80,000. He repairs it and sells it at an increase of 10%. What is the selling price of the car?
- 4. The monthly consumption of milk in a family is 120 litres. If due to festival the consumption of milk increased by 20%, find the present consumption of milk.
- 5. The cost of a saree was ₹ 500. If the shopkeeper allows a discount of 20% on the saree, what is the cost of saree after discount?

### **PROFIT AND LOSS**

Do you remember Cost Price and Selling Price?



#### PROFIT OR LOSS PER CENT AND ITS APPLICATION

Example 5: Mohan bought a pen for ₹ 5 and sold it for ₹ 7. What is the profit per cent?

**Solution:** Cost price of the pen = ₹ 5

Selling price of the pen = ₹7

Profit = ₹ 7 – ₹ 5

By spending ₹ 5, Mohan has made a profit of ₹ 2.

$$\therefore \qquad \text{Profit } \% = \frac{\text{Profit}}{\text{CP}} \times 100$$
$$= 40\%$$

Example 6: Rahul bought a house for ₹ 2, 80, 000 but due to some reason he sold the house for ₹ 2, 32,000. Find his loss and loss per cent.

**Solution:** Cost price of house = ₹ 2,80,000

Selling price of house = ₹ 2,32,000

$$Loss \% = \frac{Loss}{CP} \times 100$$

$$\therefore Loss per cent = \left(\frac{48000}{280000} \times 100\right)$$

$$= 17 \frac{1}{7}\%$$
Remember
Profit per cent or Loss per cent is always calculated on CP.

#### Worksheet 3

1. Find gain or loss per cent.

(a)	CP = ₹ 900	SP = ₹ 1,080
(b)	CP = ₹ 2,400	SP = ₹ 2,250
(c)	CP = ₹ 750	SP = ₹ 8,50

- SP = ₹ 8,670 (d) CP = ₹ 10,200
- 2. A shopkeeper buys 30 chocolate bars at ₹ 15 each and sold all of them for ₹ 525. Find the profit per cent he made.
- 3. A man buys a radio set for ₹ 900 and sells it for ₹ 972. What is his profit or loss per cent?
- 4. Mr. Gupta purchases a house for ₹ 2,50,000 and spends ₹ 50,000 on repairs. If he sells it for ₹ 4,00,000, find his gain per cent.

### **SIMPLE INTEREST**

#### Do you remember the three factors determining simple interest?

- (i) Principal (P)
- (ii) Rate of Interest (R) {in per cent}
- (iii) Time (T)

Simple Interest = Principal × Time × Rate of Interest

Simple Interest when added to the Principal gives Amount.

Amount = Simple Interest + Principal

Example 7: Vaibhav deposits ₹ 3,000 is a bank for a period of two years. If the bank gives an interest of 5% per annum, find the amount Vaibhav would get back at the end of two years.

Solution:

Principal = ₹3,000

Period (Time) = 2 years

Rate of interest = 5% per annum

To find the amount, we have to find the Interest.

SI = P × R × T  
SI = 
$$\frac{3000 \times 2 \times 5}{100}$$
 = ₹ 300  
Amount = SI + P  
= ₹ 300 + ₹ 3,000  
= ₹ 3,300

Vaibhav will get back an amount of ₹ 3,300 at the end of two years.

#### Worksheet 4

#### 1. Fill in the following blanks.

- Principal = ₹ 600, Time = 2 years, Rate of Interest = 10% p.a.
   Simple Interest = \_\_\_\_\_.
- (b) Principal = ₹ 6,500, Time =  $1\frac{1}{2}$  years, Rate of Interest = 10% p.a. Simple Interest = \_\_\_\_\_.
- (c) Principal = ₹ 1,280, Time =  $2\frac{1}{2}$  years, Rate of Interest =  $2\frac{3}{4}$ % Simple Interest = \_\_\_\_\_, Amount = \_\_\_\_.
- (d) Principal = ₹ 2,600, Time = 3 years, Rate of Interest =  $5\frac{1}{2}$ % Simple Interest = \_\_\_\_\_, Amount = \_\_\_\_.
- 2. Hari borrowed ₹ 450 from his friend at 6% per annum. He returned the amount after eight months. How much money did he pay?
- 3. Find the simple interest on ₹ 2,000 for six months at the rate of  $4\frac{1}{2}$ % per annum.

- 4. A farmer borrowed ₹ 5,300 at 8% interest per annum. At the end of  $2\frac{1}{2}$  years, he cleared his account by paying ₹ 4,600 and a watch. Find the cost of watch.
- 5. Harish borrows ₹ 2,500 at 3% per annum and ₹ 1,000 at 5% per annum interest. What interest will Harish have to pay after two years?
- Rohan deposited ₹ 5,000 at 8% per annum for 3 1/2 years and Sunil deposited ₹ 5,000 at 6% per annum for four years. Who will get more interest? What amount will each get?

#### SIMPLE INTEREST WHEN TIME IS GIVEN IN DAYS

When time is given in days, we convert the time in years and then follow the same formula for calculating simple interest.

Example 8: Find the Simple Interest on a sum of ₹ 2000 at 5 1/2 % per annum from 7 June 2015 to 19 August 2015.

Solution: Princi

Principal = ₹ 2000

Rate of interest = 
$$5\frac{1}{2}\% = \frac{11}{200}$$

Time = 7 June 2015 to 19 August 2015

Let us calculate the number of days from 7 June to 19 August 2015

June → 23 days (30 – 7)  
July → 31 days  
August → 19 days  
Time = (23 + 31 + 19) days  
= 73 days = 
$$\frac{73}{365}$$
 years  
=  $\frac{1}{5}$  years (Table of 73)  
Simple Interest =  $\frac{2000 \times 11}{200} \times \frac{1}{5}$   
= ₹ 22

#### Worksheet 5

- 1. Find the simple interest on ₹ 1300 at the rate of 5% per annum for 146 days.
- 2. Find the amount Meena gets on depositing ₹ 1500 at 8% per annum for 73 days.
- 3. What amount will Reena get by investing ₹ 4500 at  $4\frac{1}{2}$ % per annum for the period from September 8 to November 20 of the year 2015.
- 4. Find the simple interest on ₹ 3500 at  $4\frac{1}{2}$ % per annum from 15 March to 8 August 2015.

## ALUE BASED QUESTIONS

- An NGO (Non-Government Organisation) working for the flood relief fund collected a sum of ₹ 80000 from different societies of a locality. 65% of this amount was spent for buying grocery items. Rest of the amount was spent on clothes for the affected people. The flood affected victims were very thankful to the NGO members.
  - (a) What amount was spent on grocery items?
  - (b) Write any two more ways in which you can help the flood affected victims.
- During winter season, the Manav Seva Samiti decided to distribute blankets and shawls to some poor people of a locality. A total of 150 packets were distributed. 74% of the poor people received shawls and the others got blankets. The poor people were very happy with the Shawls and blankets.
  - (a) How many blankets were distributed?
  - (b) Name two more items that you have donated to any poor person.

## **BRAIN TEASERS**

- 1. A. Tick ( $\checkmark$ ) the correct answer.
  - (a) 2.6% expressed as a decimal is-
    - (i) 0.26 (ii) 0.026 (iii) 26 (iv) 2.6

What per cent is  $\frac{1}{2}$  dozen of one score? (b) (i) 10% (ii) 60% (iii) 30% (iv) 25% If CP = ₹ 75, SP = ₹ 100, then gain or loss per cent is-(c) (i) Gain of  $33\frac{1}{3}\%$ (ii) Loss of  $33\frac{1}{3}\%$ (iii) Loss of 25% (iv) Gain of 25% (d) Gain or Loss per cent is always calculated on-(i) SP (ii) Loss (iii) Gain (iv) CP (e)  $33\frac{1}{3}\%$  of  $1\frac{1}{2}$  minute is equal to-90 seconds 30 seconds (i) (ii) (iii) 50 seconds 60 seconds (iv)

#### B. Answer the following questions.

- (a) What is the number which is 5% more than 600?
- (b) Find the simple interest on ₹ 200 at 5% per annum for 6 months.
- (c) If 140 is reduced to 112, what per cent is the reduction?
- (d) What is 12% of 0.5 metre?
- (e) If SP = ₹ 260, Profit = ₹ 60, find profit per cent.
- 2. Find the value of-
  - (a)  $33\frac{1}{3}\%$  of 9012
  - (b) 25% of 10% of 1 kg
- 3. Find  $12\frac{1}{2}\%$  less than 16 hours.
- 4. John bought 100 eggs for ₹ 40. Out of these, four eggs were found to be broken and he sold the remaining eggs at the rate of ₹ 7.50 per dozen. Find his gain or loss per cent.

- 5. Articles are bought at ₹ 45 per dozen and sold at ₹ 85 per score. Find the gain or loss per cent.
- 6. Salim deposited ₹ 12,000 in a finance company which pays 15% interest per year. Find the amount he is expected to get after  $4\frac{1}{2}$  years.
- Ramlal bought oranges at ₹ 30 per dozen. He had to sell them at a loss of 5%. Find the selling price.
- 8. Out of 1,200 people, 800 know only English, 50 know only Punjabi and the rest know both languages. Find the per cent of—
  - (a) people who know only English.
  - (b) people who know both English and Punjabi.
- 9. Prabal deposited ₹ 5,000 in a bank which pays him 5 1/2% interest. After three years, he withdraws the money and buys an almirah for ₹ 4,700. How much money is left with him?
- 10. Find the Simple Interest on ₹ 6500 at 8% from 5 January 2015 to 19 March 2015.

## HOTS

- 1. The selling price of an article is  $\frac{4}{3}$  of its cost price. Find the profit per cent.
- 2. In a school there are 50 students in class VI A. 88% of the students passed a Mathematics test. The same number of boys and girls failed in the exam. If 42% of the total students are girls, how many boys passed the test?
- 3. The cost price of 10 articles is equal to the selling price of 8, find the profit or loss per cent.
- 4. Find the number by adding  $33\frac{1}{3}\%$  of  $\frac{1}{3}$  of the predecessor of the smallest 5-digit number to the successor of the smallest number formed by the digits of first four composite numbers.

## **ENRICHMENT QUESTION**

#### Complete the given magic square.

$\frac{8}{5}$	30%	$\frac{2}{10}$	1.3
	1.0		80%
1.1	0.6		1.2
40%	1.5	$\frac{7}{5}$	

## YOU MUST KNOW

- 1. A fraction with denominator equal to 100 is called a percentage.
- 2. To convert a given quantity into percentage multiply the quantity by 100 and put the symbol '%'.

e.g. 
$$\frac{2}{5} = \left(\frac{2}{5} \times 100\right)\% = 40\%$$

3. We can find percentage of any quantity.

e.g. 24% of 50 marbles = 
$$\left(\frac{24}{100} \times 50\right)$$
 = 12 marbles.

4. Profit or Loss per cent is calculated on CP.

Profit per cent = 
$$\left(\frac{\text{Profit}}{\text{CP}} \times 100\right)$$
  
Loss per cent =  $\left(\frac{\text{Loss}}{\text{CP}} \times 100\right)$ 

- 5. The three factors determining the Simple Interest are Principal, Rate of interest and Time.
- 6. Simple Interest = Principal  $\times$  Time  $\times$  Rate of Interest.
- 7. Amount = Simple Interest + Principal.
- 8. If time is given in months or days, convert the time into years and then find the Simple Interest.

**INTRODUCTION TO ALGEBRA** CHAPTER 6 INTRODUCTION Perimeter of this square garden is What is the perimeter  $4 \times 6m = 24 m$ of this square garden? We have, Perimeter =  $4 \times \text{side}$ Ι. or simply  $p = 4 \times s$ II. Similarly, area of this rectangle is 4 cm Area =  $4 \text{ cm} \times 2 \text{ cm} = 8 \text{ sq. cm}$ 2 cm Area = length  $\times$  breadth or simply  $a = I \times b$ III. If the cost price is ₹ 10 and selling price is ₹ 14, we have a gain of ₹ 14 – ₹ 10 = ₹ 4 Gain = SP - CPg = s - cor simply In the above three cases, we have used letters to represent numbers.  $p = 4 \times s$ ,  $a = I \times b$  and g = s - c

When letters of alphabet represent numbers, they are called Literal Numbers.

#### Do you know?

- The word Algebra is derived from the title of the book, Algebar W' al-almugatalah written by an Arab mathematician, Mohammed Ben Musa.
- Indian mathematician, Aryabhatta is said to have introduced Algebra in India.
- The first person to develop algebraic method of problem solving was the French mathematician, **Francois Viete.** He is also called the "Father of Algebra".

#### Let us look at some examples.

**Example 1:** Write the following using numbers, literals and arithmetic operations.

- (a) 7 less than x
- (b) 3 more than quotient of x by y
- (c) 5 times x added to 2 times y

#### Solution:

So, we write x - 7

(b) Quotient of x by y can be written as  $\frac{x}{y}$ 

(a) 7 less than x means 7 is subtracted from x.

3 more than  $\frac{x}{y}$  is  $\left| \frac{x}{y} + 3 \right|$ 

(c) 5 times x is 5x, 2 times y is 2y,

5 times x added to 2 times y is 5x + 2y

**Example 2:** I had a 10 metre long ribbon. I used *x* metres of it for my doll. How much ribbon is left with me?

**Solution:** Length of ribbon I had = 10 m

Length of ribbon used = x metres

Length of ribbon left = (10 - x) metres.

#### Worksheet 1

- 1. Write the number which is-
  - (a) 5 more than x
  - (b) 3 less than y
  - (c) 2 times z

(d) One-third of x

3.

- (e) 4 more than sum of *a* and *b*
- 2. Write the following expressions using numbers, literal numbers and arithmetic operations.

(a)	x added to y	
(b)	<i>m</i> increased by 2	
(c)	Sum of $x$ , $y$ and $z$	
(d)	2 times <i>p</i> added to 3 times <i>q</i>	
(e)	One-third the sum of a and b	
(f)	4 less than x	
(g)	x less than 4	
(h)	p taken away from twice q	
(i)	3 times x	
(j)	One-fifth of $y$ added to $x$	
(k)	5 times the sum of $x$ and $y$	
(I)	a divided by 6	
(m)	Quotient of x by 5	
(n)	Half the sum of <i>p</i> and <i>q</i>	
Wri	te the following expressions in words.	

- (a) x + 5 (b) y 3 (c)  $\frac{2}{3}x$ (d) 4z (e)  $\frac{x}{y}$  (f) 2a + 3b
- 4. There are 38 students in a class. If x more students joined the class, how many students are there in the class?
- 5. Three sides of a triangle are x cm, y cm and z cm. Find the perimeter of the triangle.
- 6. Cost of one pen is  $\gtrless$  8. Find the cost of y pens.
- 7. Neha has seven more toffees than Megha. If Megha has *x* toffees, how many toffees does Neha have?
- 8. Anshul is z years old. How old was he three years ago?

#### **RULES AND PROPERTIES OF OPERATIONS**



#### Worksheet 2

Fill in the following blanks using properties.

- 2x + y = + 2x1. 2. 4a + 2c = 5b + 4a 7.  $2b \times 2b = 2b$
- 3. 5y 0 =.
- 5. 2*p* ÷ 1 =
- $6. \qquad \div x = 0$ 8.  $(-4x) \times 3y = 3y \times$ 4.  $(3x) \times (2y) = 2y \times$  9.  $x \times (2y + z) = x \times$   $+ x \times$ 10. (3p + 2q) + r = 3p + ( +

## **ALGEBRAIC EXPRESSIONS**

#### A. POWER OF LITERAL NUMBERS



Volume of a cube of edge 'x' cm is Volume =  $(x) \times (x) \times (x)$  cubic cm See! area of this square of side 'a' cm is Area =  $(a \times a)$  square cm



We can also write

In $a^2 \leftarrow a^2 \leftarrow$ Exponent or power Base	$a \times a$ as $a^2$
In $x^3 \leftarrow X^3 \leftarrow$ Exponent or power Base	$x \times x \times x$ as $x^2$

We write	We read	
a <sup>2</sup>	'a' square or a to the power 2	
X <sup>3</sup>	'x' cube or x to the power 3	
X <sup>4</sup>	x to the power 4	
$(-x)^5$	-x to the power 5	

Note:

In a literal number 'a', base is 'a' and power is 1 because  $a = a^1$ 

Remember  

$$(2x)^3 = 2x \times 2x \times 2x$$
  
 $2x^3 = 2 \times x \times x \times x$ 

#### **B. VARIABLE AND CONSTANT**



#### Observe this table.

Radius	Diameter	
5 cm	$2 \times 5 = 10 \text{ cm}$	
3.6 cm	<b>2</b> × 3.6 = 7.2 cm	
10 cm	<b>2</b> × 10 = 20 cm	



- A quantity which does not change its value is called a Constant.
- A quantity which changes its value is called a Variable.

#### Remember

The number of hours in a day, the number of months in a year, the number of seconds in an hour are all constants because they always remain the same. But, the length and breadth of a rectangle, the side of a square may vary as per their size. These are all variables.

#### **C. COEFFICIENT**



- The coefficient of x in x is 1 because  $x = 1 \times x$
- The coefficient of x in -x is -1 because  $-x = -1 \times x$

#### **D. NUMERICAL COEFFICIENT**

Let us take 4x.



#### 1. Write the exponential form of-

- (a)  $(y) \times (y) \times (y) \times (y) \times \dots$  9 times
- (b)  $p \times p \times p \times \dots$  21 times

(c) 
$$\frac{1}{2} \times a \times b \times b \times c$$
  
(d)  $2 \times a \times a \times a \times b \times b \times c$   
(e)  $-6 \times (x) \times (y) \times (y) \times (z)$   
(f)  $(-z) \times (-z) \times (-z) \times (-z)$   
(g)  $(-a) \times (-a) \times (-a) \times \dots 17$  times  
(h)  $(3x) \times (3x) \times (3x) \times (3x) \times (3x)$   
2. Write the product form of the following:  
(a)  $y^{6}$  (b)  $x^{19}$  (c)  $\frac{3}{4}x^{2}yz$  (d)  $a^{3}b^{2}c$   
(e)  $(-a)^{4}$  (f)  $(2x)^{3}$  (g)  $(-z)^{30}$  (h)  $-7pq^{2}r^{3}$   
3. Write the product form of  $(3x)^{3}$  and  $3x^{3}$ . Are they same?  
4. Write the coefficient of-  
(a)  $x \ln 2x$  (b)  $a \ln - 9a$  (c)  $y \ln 4xy$  (d)  $b \ln - 8abc$   
(e)  $a \ln - a$  (f)  $z \ln \frac{4}{3}z$  (g)  $x^{2} \ln 4x^{2}y$  (h)  $y \ln y$   
(i)  $a \ln - \frac{1}{2}ab^{2}$  (j)  $z \ln zy$   
5. Identify the following statements as constants or variables.  
(a) Number of mangoes in one dozen mangoes.  
(b) Number of centimetres in a metre.  
(c) Temperature in a day.  
(d) Degrees in an acute angle.  
(e) Degrees in an acute angle.  
(f) Number of days in February.  
(g) Number of letters in the word MATHEMATICS.  
6. Write the numerical coefficient in-  
(a)  $2x$  (b)  $3yz$  (c)  $\frac{1}{4}x^{2}z$  (d)  $a$   
(e)  $-yz$  (f)  $3pqr$  (g)  $\frac{4}{3}a^{2}bc$  (h)  $x^{2}yz$ 

#### E. ALGEBRAIC EXPRESSIONS

We know  $3 + 6 \div 2 - 6$  is a numerical expression.

Numerical expression is a combination of numbers and fundamental operations.

See this expression  $\rightarrow 4x + 2y - 5z$ Literal numbers 4, 2, 5 are numbers

A combination of numbers, literal numbers and fundamental operations is called an **Algebraic Expression**.

$$5a^{2} + 2b^{2} - c^{2}$$
  
- 2abc  
$$x + y - z + 3$$
  
-  $\frac{1}{2}x + \frac{2}{z}y + 5$  are all algebraic expressions.

In the algebraic expression

 $3x + 2y - 5 \rightarrow 3x$ , 2y and 5 are the parts separated by '+' and '-' signs.

Parts of an algebraic expression separated by the symbols (+) and (-) are called the **Terms** of the algebraic expression.

### LIKE TERMS AND UNLIKE TERMS

See the variables in these terms.



Terms having the same variables are called Like Terms.

Terms 7x, -3y,  $\frac{1}{5}z$ ,  $3x^2$  have different variables.

Terms having different variables are called Unlike Terms.

Expression	Number of Terms	Name of Expression
3 <i>x</i>	1	Monomial
– 2a + 3b	2	Binomial
2p - 3q + 7	3	Trinomial
x - 2y - z + 8	4	Quadrinomial

Now, let us learn about the names of expressions.

Example 3:



#### Worksheet 4

1. Separate the terms of the following algebraic expressions.

(a)	$x^2 - 2xy + y^2$	(b)	$5a^2 + 6ab - 2b^2 + 3$
(c)	$p^3 + q^3 + r^3 - 3pqr$	(d)	x + y - z - xyz + 5
(e)	$5x^2y - 5xy^2$	(f)	$2x - 3y^2 + \frac{1}{5}x^2 - \frac{2}{3}y$

2. Classify each of the following as monomial, binomial, trinomial or quadrinomial expressions.

(a)	X + Y	(b)	3
(c)	$a^2 + 2ab + b^2$	(d)	$\frac{1}{4}x$
(e)	p-q+r-s	(f)	– 2a + 3b
(g)	3 <i>xyz</i>	(h)	0
(i)	5xy + 2yx + 3	(j)	-pq + qr + rp - pqr
(k)	4 <i>abc</i> – 2	(I)	$X^3 + X^2 - X$

3. Is 4x + 3y + 2x a trinomial expression? Why not?

Identify the like terms in the following 4.

- (a)  $x^2$ ,  $y^2$ ,  $z^2$ ,  $\frac{1}{2}x^2$ (b) p, 3q, r, 4p
- (c) 2*ab*, 5*a*, 3*ba*, *bac* (d)  $x^2yz$ ,  $2x^2y^2z$ ,  $4xyz^2$ ,  $-4yx^2z$
- (f) -a, 2b, 7a, 3c, a (e) – 11*pq*, 7*pr*, *qp*, 4*rq*
- (g)  $3a^2b^2c$ ,  $2a^3b^2c$ ,  $2c^3ba^3$ ,  $b^2a^3c$ (h)  $4x^2y^2$ ,  $-2x^2y$ , 3xyz,  $2yx^2$
- 5. Write the algebraic expression whose terms are-
  - (b)  $4x^2y_1 2xy^2_1 3$ (a) 2a, 3b, - 2c
  - (c)  $x_{1} y_{2} z_{3} 3$
- Write two examples of each expression. 6.
  - (a) Binomial expression
  - (b) Monomial expression
  - (c) Trinomial expression

#### **OPERATIONS ON ALGEBRAIC EXPRESSIONS**

#### A. ADDITION OF ALGEBRAIC EXPRESSIONS

We know,









4 apples + 3 bananas

See! we can simply write

4 apples

4a + 3b

3 bananas

(Taking apple as 'a' and bananas as 'b')

=



Let us add some algebraic expressions.

#### I. ADDITION OF MONOMIALS

Example 4: Add 2x, 3x, 6x  
Solution: 
$$2x + 3x + 6x = (2+3+6)x$$
 Adding numerical coefficient of each monomial.  
 $= 11x$  Variable is the same.  
Example 5: Add 4a, 3b, - 2a, 6b, 5a  
Solution:  $4a + 3b - 2a + 6b + 5a$  (We have unlike terms)  
 $= (4a - 2a + 5a) + (3b + 6b)$  (Regrouping the like terms)  
 $(4-2+5)a \xrightarrow{7a + 9b} (3+6)b$ 

#### **II. ADDITION OF BINOMIALS**

**Example 6:** Add 5a + 2b and 3a - 12b

Solution: Addition can be performed by two methods.

- 1. Column method
- 2. Horizontal method

#### Column method



#### Horizontal method

$$(5a + 2b) + (3a - 12b)$$
  
=  $(5a + 3a) + (2b - 12b)$   
=  $8a - 10b$ 

(Rearranging like terms) (Adding numerical coefficients)

#### **III. ADDITION OF TRINOMIALS**

**Example 7:** Add  $-4x^2 + y^2 - 3$ ,  $5y^2 + 2x^2 + 5$ ,  $2x^2 - 8y^2$ .

Solution: We can use two methods:

#### Column method



#### Horizontal method

$$(-4x^{2} + y^{2} - 3) + (2x^{2} + 5y^{2} + 5) + (2x^{2} - 8y^{2})$$

$$= (-4x^{2} + 2x^{2} + 2x^{2}) + (y^{2} + 5y^{2} - 8y^{2}) + (-3 + 5)$$
(Regroup the terms as per their variables.)
$$= \underbrace{0x^{2} + 2y^{2} + 2}_{4}$$
Anything multiplied by zero (0) is zero. So.  $0x^{2}$  is omitted.
$$= -2y^{2} + 2$$

#### Worksheet 5

- 1. Add the following monomials.
  - (a) 3x, 5x, 2x (d) -2xy, 7xy, 15xy, -10xy, xy, -xy
  - (b) 5a, a, -7a (e) 3a, 4b, -6a, -2b, -5a, 10b
  - (c) 2abc, -abc, 5abc (f) a, 6a, -2b, 5c, -2c
- 2. Find the sum using column method.
  - (a) 7a + 2b and 3a + 4b
  - (b)  $2x^2 y^2$  and  $3x^2 + 5y^2$
  - (c) -4x 5y and 3x 8y
  - (d) 4x + 3y + 5xy, 2x + 10y 2xy and -3x 3y
  - (e) -2a + 3b, 5a + 2b c and -a b c
  - (f)  $3x^2 + 4y^2$ ,  $2y^2 + 2xy x^2$ ,  $x^2 + 2y^2$

- 3. Add using horizontal method.
  - (a) 7x + 3y and -4x 2y
  - (b) 3a 2b + c + 3 and a + 3b 2c 4
  - (c)  $a^2 + 2ab + b^2$  and  $a^2 2ab + b^2$
  - (d) xy yz + 2; 2yz + xy 7 and 3xy + 3yz + 3
  - (e)  $-2x^2 + 3y^2$ ;  $3y^2 + 3 x^2$  and  $x^2 + y^2 + 5 3xy$
  - (f) 3pq + 2pr 4qr; -pq + 2qr + pr and 4pq 3pr + 2qr

#### **B. SUBTRACTION OF ALGEBRAIC EXPRESSIONS**



5a - 3a = 2a (apple is denoted by literal 'a')

We cannot subtract 3 bananas from 5 apples.



5 apples

3 bananas

So, we can simply write 5a - 3b (Taking apple as 'a' and banana as 'b')



Let us subtract some algebraic expressions.

#### I. SUBTRACTION OF MONOMIALS

- **Example 8:** Subtract 3*x* from 7*x*
- Solution: Here also we can use two methods:

#### Column method



#### Horizontal method

7x - 3x = 7x + (-3x) We add the negative of 3x to 7x = 4x

#### **II. SUBTRACTION OF BINOMIALS**

**Example 9:** Subtract  $-3a^2 + 2b^2$  from  $-4a^2 + 3b^2$ 

Solution: We can use two methods:

#### **Column method**

 $-4a^{2} + 3b^{2}$  (We write the minuend)  $-3a^{2} + 2b^{2}$  (We write the subtrahend) + - (We change the signs of the terms in subtrahend and then add)  $-a^{2} + b^{2}$  (3-2) $b^{2}$ 

#### Horizontal method

$$(-4a^{2} + 3b^{2}) - (-3a^{2} + 2b^{2})$$

$$= -4a^{2} + 3b^{2} + 3a^{2} - 2b^{2}$$
 (We change the sign of the terms in the subtrahend)
$$= (-4a^{2} + 3a^{2}) + (3b^{2} - 2b^{2})$$
 (We group like terms)
$$= -a^{2} + b^{2}$$
 (We add the like terms)

#### **III. SUBTRACTION OF TRINOMIALS**

We subtract trinomials in the same way as we subtract binomials.

Example 10: Subtract  $3a^3 + 2b^3 - 3ab$  from  $-2a^3 - 4b^3 + ab$ Solution: Here,  $-2a^3 - 4b^3 + ab$   $3a^3 + 2b^3 - 3ab$   $- 5a^3 - 6b^3 + 4ab$  (We change the signs of terms) (We add like terms)

**Example 11:** From the sum of  $3x^2 + 2y^2$ ,  $-x^2 - 4y^2 + 2xy$  and  $6xy + 2x^2 - 3y^2$ ,

subtract  $-5x^{2} + 2y^{2} - 3xy$ .

#### Solution: First we add

$$3x^{2} + 2y^{2}, -x^{2} - 4y^{2} + 2xy \text{ and } 6xy + 2x^{2} - 3y^{2}.$$
  

$$3x^{2} + 2y^{2} - x^{2} - 4y^{2} + 2xy + 6xy + 2x^{2} - 3y^{2}$$
  

$$= (3x^{2} - x^{2} + 2x^{2}) + (2y^{2} - 4y^{2} - 3y^{2}) + (2xy + 6xy)$$
  

$$= 4x^{2} - 5y^{2} + 8xy$$

Now, we subtract

$$-5x^{2} + 2y^{2} - 3xy \text{ from } 4x^{2} - 5y^{2} + 8xy$$

$$(4x^{2} - 5y^{2} + 8xy) - (-5x^{2} + 2y^{2} - 3xy)$$

$$= 4x^{2} - 5y^{2} + 8xy + 5x^{2} - 2y^{2} + 3xy \quad (\text{Sign of terms changed})$$

$$= (4x^{2} + 5x^{2}) + (-5y^{2} - 2y^{2}) + (8xy + 3xy) \quad (\text{Like terms are grouped})$$

$$= 9x^{2} - 7y^{2} + 11xy \quad (\text{We add like terms})$$

## Worksheet 6

1. Subtract.

(a)	3x from 5x	(b)	4 <i>a</i> from 3 <i>a</i>
(c)	$13x^{2}$ from $x^{2}$	(d)	– y from 3y

(e) -7a from -3a (f) 3abc from -3abc

- 2. Subtract by column method.
  - (a) 3x + 2y from 2x + 3y
  - (b)  $a^2 + 2b^2$  from  $3b^2 4a^2$
  - (c)  $5x^2y 3xy^2$  from  $-2x^2y + 4xy^2$
  - (d) 2a 3b + 5 from 5a b 2
  - (e) p + q r from -p q r
  - (f)  $a^3 + b^3 3abc$  from  $-a^3 + b^3 + 3abc$
- 3. Subtract by horizontal method.
  - (a) 2x 3y + 4z from 4x 6y z
  - (b) 2x 4y from -4x + 3y
  - (c)  $5q^2 p^2 7r$  from  $6p^2 q^2 + 2r 9$
  - (d) -x y from x + y
  - (e) 7pqr 8p + 3 from 10 4pqr + 3p
  - (f)  $5x^3 + 3x^2 x 3$  from  $5x^3 3x^2 + 2x 3$
- 4. Subtract 3x + 2y - 5xy from 0
- From the sum of  $x^2 + x + 1$  and  $x^2 x + 1$ , subtract  $-x^2 x + 1$ 5.
- 6. Subtract the sum of 2a + 3b, a - 2b + c, -a + 2c and 4a + 2b - 5 from 6a - 4b + 8

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#### FINDING THE VALUE OF AN ALGEBRAIC EXPRESSION

If values of literals are given, we can find the value of the algebraic expression.

Example 12: Find the value of 
$$2x + 3y + 1$$
 if  
 $x = 3$  and  $y = -2$   
Solution:  $2x + 3y + 1 = 2 \times x + 3 \times y + 1$   
 $= 2 \times 3 + 3 \times (-2) + 1$  (We substitute  $x = 3$  and  $y = -2$ )  
 $= 6 - 6 + 1$  (We multiply and then add the terms)  
 $= 0 + 1$   
 $= 1$ 

#### Worksheet 7

- 1. If a = 2, b = 3 find the value of the following algebraic expressions.
  - (a) 3a + 2b (b) 2a b
- 2. If x = -1, y = 2, z = -3, find the value of-

a) 
$$x + y + z$$
 (b)  $2x - 3y + 6z$ 

3. If p = -3, q = 0, r = 2, find the value of-

(a) 
$$p + q + r + 1$$
 (b)  $2pq + 3qr - 4pr$ 

- 4. If z = -9, find the value of  $\frac{2}{3}z + 6$
- 5. If a = 1, b = -2, c = -1, show that 7a + b + c = 2abc

## ALUE BASED QUESTIONS

- 1. Amounts donated by Mr. Lal and Mr. Sohan to an orphanage are represented by the expressions  $(2a^2 3b^2 + 6ab)$  and  $(5b^2 2ab + 3a^2)$  respectively.
  - (a) Find the expression for total amount donated by them. Also, when a = 100, b = 150 find the total amount.
  - (b) What value is depicted by them?
- 2. Number of students of Class-V and VI who participated in "SAVE ENVIRONMENT" campaign is given by the expression  $(2pq + 3p^2 + 2q^2)$  and  $(4p^2 + q^2 + 3pq)$  respectively.
  - (a) If p = 2, q = 5, find how many more students of Class-V participated in the campaign.
  - (b) What value is possessed by participating students?

## **BRAIN TEASERS**

- 1. A. Tick ( $\checkmark$ ) the correct answer.
  - (a) Mrs. Shalini is p years of age now, 3 years ago her age in years, was-
    - (i) 3 p (ii) 3 + p (iii) p 3 (iv)  $3 \div p$

- (b) If the perimeter of a square is q metres, then length of each side in metres, is-(ii)  $q \div 4$ (iiii) a-4(i) q + 4(iv)  $4 \div a$ (c) Expression obtained when k is multiplied by 2 and then subtracted from 5, is-(iii) 5 – 2*k* (i) 2k - 5(ii) 2k + 5(iv) 5k - 2Number of notebooks bought for  $\mathfrak{T}$  t at the rate of  $\mathfrak{T}$  40 per notebook, is-(d) (ii) 40 t (iii)  $\frac{t}{40}$  (iv) t + 40(i)  $\frac{40}{t}$ (e) x = 6, y = 2, z = 3, what is the value of the expression  $\frac{xy - xz}{z^2}$ ? (i)  $\frac{-2}{3}$  (ii)  $\frac{2}{3}$  (iii)  $3\frac{1}{3}$  (iv) 5 B. Answer the following questions.
  - (a) Your Mom has a sum of ₹ x. She spent ₹ 3000 on grocery and ₹ 2000 on milk.
     Write the algebraic expression for the amount left with her.
  - (b) The digit in the ones place of a 2-digit number is twice the digit at the tens place. Write the number.
  - (c) If a notebook costs ₹ p and a pencil costs ₹ s, then what will be the cost of two notebooks and three pencils?
  - (d) What will you get if 3 times x is subtracted from the largest two-digit number?
  - (e) Rohit travels by car with the speed of x km/hr for 2 hours and 3 hours with the speed of y km/hr. Write the expression for total distance travelled by him.
- 2. Write the number which is—
  - (a) 6 less than two-third of number x
  - (b) 3 times the sum of z and 5
- 3. Write statements for-

(a) 
$$\frac{4}{5}(a+2)$$
 (b)  $\frac{4}{5}a+2$ 

- 4. Write the following expressions using literals, numbers and arithmetic operations.
  - (a) diameter of a circle is twice its radius.
  - (b) twice of x subtracted from one-third of y.
  - (c) total cost of three tables at  $\overline{\uparrow} x$  each and four chairs at  $\overline{\uparrow} y$  each.

- (d) cost of one book if the cost of 10 books is  $\overline{\tau} x$ .
- 5. Write the product form of the following:

(a)  $-3 p^3 q^2 r$  (b)  $(-y)^{27}$ 

- 6. Write a statement to represent a constant.
- 7. Write a statement to represent a variable.

8. Write the product form of 
$$\left(\frac{2}{3}x\right)^3$$
 and  $\frac{2}{3}x^3$ . Are they same?

- 9. Write one example each for a—
  - (a) Monomial expression (b) Trinomial expression
  - (c) Quadrinomial expression (d) Binomial expression
- 10. Separate the terms of-

 $3p^3 - 2p^2 + p - 6$ 

11. Encircle the like terms.

a²bc,	$a^2b^2c$ ,	$b^{2}a^{2}c^{2},$	—	ba²c, c²ab,
cba².	bca².	$-a^{2}bc$		

- 12. Fill in the following blanks.
  - (a) An algebraic expression whose terms are 2*a*, 3*b*, 5 is \_\_\_\_\_.
  - (b) The numerical coefficient in  $-\frac{3}{2}x^2yz$  is \_\_\_\_\_.
  - (c) The coefficient of y in 2.5  $x^2y$  is \_\_\_\_\_.
  - (d) The coefficient of p in -p is \_\_\_\_\_.
  - (e) Is 2x 3y x a binomial or a trinomial expression? \_\_\_\_\_.
  - (f) The exponential form of  $(-p) \times (-p) \times (-p) \times ....17$  times is \_\_\_\_\_.
  - (g) Sum of -7x, 8x, x, -9x is \_\_\_\_\_.
  - (h) If we subtract -9y from -2y, we get \_\_\_\_\_.
- 13. Evaluate a 4b + 2 if a = 2, b = 1
- 14. In a bus x passengers have taken, tickets of  $\stackrel{?}{<}$  5 each and y passengers have taken tickets of  $\stackrel{?}{<}$  10 each. Find the total number of passengers and total fare received by the conductor.

15. On a particular day x ladies were travelling in ladies compartment of metro rail and y people in other compartments from Station A to Station B. If 10 passengers from other compartments and 15 passengers (ladies) from ladies compartment got down in between, write the algebraic expression for the number of passengers left at Station B.

## HOTS

If two cubes whose volumes are given by  $(x^3 + 6y^2 + 12z + 8)$  cubic units and  $(x^3 - 2z + 10 - y^2)$  cubic units respectively are melted to form a new cube, find the expression representing the volume of the new cube thus formed.

## YOU MUST KNOW

- 1. Literal numbers or literals are the letters of alphabets used to represent numbers.
- 2. Literal numbers obey all the rules regarding the operations of addition, subtraction, multiplication and division, i.e.

$$\begin{cases} x + y = y + x \\ x + 0 = x \\ x + (y + z) = (x + y) + z \end{cases}$$
Addition  
$$\begin{cases} x - y \neq y - x \\ x - 0 = x \end{cases}$$
Subtraction  
$$\begin{cases} xy = yx \\ x(y + z) = xy + xz \\ x \times 1 = x \\ x \times (0) = 0 \\ x \div x = 1 \\ x \div 1 = x \end{cases}$$
Multiplication  
$$\begin{cases} x + y = y + xz \\ x + z = x \\ x + z = 1 \\ x \div 1 = x \end{cases}$$
Division

- 3. For any literal number x, base is x and power is 1 (as  $x = x^{1}$ ).
- 4. Constant is a quantity which does not change its value.
- 5. Variable is a quantity which changes its value.
- 6. Algebraic Expression is a combination of numbers, literal numbers and fundamental operations.

- 7. Parts of an algebraic expression separated by the symbols (+) and (-) are called the terms of the algebraic expression. Terms having same variables are called like terms and terms having different variables are called unlike terms.
- 8. Monomials are algebraic expressions with one term. Binomials are algebraic expressions with two terms. Trinomials are algebraic expressions with three terms. Quadrinomials are algebraic expressions with four terms.
- 9. For addition and subtraction of algebraic expression only like terms are to be added or subtracted in both horizontal method and column method.

## CHAPTER

## LINEAR EQUATIONS

### **INTRODUCTION**

Look at these situations.

Situation 1.



Situation 2. Similarly, there were 18 mangoes in a basket. Some mangoes were rotten which were removed. Now there are 15 mangoes.



These mathematical statements 5 + x = 9 and 18 - x = 15 have the sign of **equality (=)**. These are called **equations**.

A statement of equality which contains literal numbers (variable) is called an **Equation**.

#### Look at these equations.



An equation in which the power of the literal number (variable) is one is called a **Linear Equation.** 

#### Every equation consists of two parts.

3x + 2	=	9
Expression on Left Hand Side		Expression on Right Hand Side
LHS		RHS

#### Observe the following table.

Linear Equation	LHS	RHS	
x - 5 = 3	<i>x</i> – 5	3	
$\frac{y}{3} = 9$	$\frac{y}{3}$	9	
3a + 2 = 5 - a	3 <i>a</i> + 2	5 – <i>a</i>	
2(x + 3) = 10	2(x + 3)	10	

### CONVERTING A STATEMENT INTO AN EQUATION

#### Let us see some examples.

**Example 1:** 4 added to a number *x* is equal to 9

(is equal to) We write 4 + x = 9Example 2: 3 times a number y is equal to 12 We write 3y = 12(is equal to)

#### Worksheet 1

- 1. What are the two sides of an equation called?
- 2. In a linear equation, what is the power of the variable?
- 3. Encircle the linear equations in the following:

(a) 
$$p + 3 = 5$$
  
(b)  $2x + 3 = 7$   
(c)  $x^2 + 1 = 5$   
(d)  $\frac{x}{3} = 7$   
(e)  $x^3 - 1 = 9$   
(f)  $2x - 5 = x - 3$ 

4. Separate the LHS and RHS of the following equations.

Linear Equation	LHS	RHS
(a) $y - 3 = 9$		
(b) $2x + 1 = 5$		
(c) $\frac{y}{3} = 9$		
(d) $5y - 1 = 9 - 2y$		

#### 5. Convert the following statements into an equation.

- (a) 3 added to a number *x* is 5.
- (b) A number *y* increased by 6 is 9.
- (c) 2 subtracted from a number *x* is equal to 10.
- (d) Twice a number *p* equals to 12.
- (e) 15 less than a number in *a* is 7.
- (f) Two-third a number x is 5.
- (g) Three times a number *x* decreased by 1 is 4.
- (h) Six times a number *y* is 3 more than the number itself.
- (i) 5 less than one-third a number z is 5.
- (j) Two times the sum of a number *x* and 3 equals 14.

## SOLVING LINEAR EQUATION

Finding the value of the unknown (literal) in an equation is called **Solving an Equation**.

#### A. SOLVING AN EQUATION BY TAKING DIFFERENT VALUES

Let us take the equation 2x = 10

We go on checking by taking different values for *x*. Observe the table.

X	2 <i>x</i>	<b>Condition Solution Yes/No</b>
1	2	No
2	4	No
3	6	No
4	8	No
5	10	Yes
6	12	No
7	14	No

LHS becomes equal to RHS only when x = 5

So, x = 5 is the solution of the equation 2x = 10

#### Worksheet 2

Check whether given variable can solve the equation. Write 'Yes' or 'No'.

	Equation	Value of the Variable	Yes/No
(a)	10 <i>y</i> = 80	<i>y</i> = 6	
(b)	<i>b</i> + 5 = 9	<i>b</i> = 4	
(c)	p - 8 = 5	<i>p</i> = - 13	
(d)	$\frac{y}{3} = -1$	<i>y</i> = - 3	
(e)	3 <i>z</i> = 12	<i>z</i> = 2	
(f)	<i>a</i> – 4 = 5	<i>a</i> = 1	
(g)	<i>a</i> + 4 = 5	<i>a</i> = – 1	
(h)	2x + 1 = 7	<i>x</i> = 3	
(i)	2x - 1 = -9	<i>x</i> = 4	
(j)	3 <i>x</i> = - 15	<i>x</i> = - 5	

#### **B. SOLVING AN EQUATION (SHORT METHOD)**

Let us solve the equation x + 4 = 11



To find the value of *x*, we have to eliminate 4 from LHS.



(But see, the balance is disturbed.)

So we have to eliminate an equal number from the RHS also.



- Add the same number to both the sides of the equation.
- Subtract the same number from both the sides of the equation.
- Multiply both sides of the equation by the same number. (non-zero)
- Divide both sides of the equation by the same number. (non-zero)

#### Note:

Whatever you do to one side, you should do the same to the other side also.

#### See more examples.

**Example 3:** Solve y - 3 = 5y-3+3=5+3Adding + 3 (the additive inverse of - 3) to both sides Solution: y + 0 = 8y = 8y = 8 is the solution for the equation y - 3 = 5**Example 4:** Solve 5y = 20 $---5y = 5 \times y$  $\frac{5y}{5} = \frac{20}{5}$  Divide both sides by 5 Solution: y = 4y = 4 is the solution of the given equation. *.*.. **Example 5:** Solve  $\frac{x}{4} = 9$ **Solution:**  $\frac{x}{4} \times 4 = 9 \times 4$  Multiply both sides by 4 x = 36x = 36 is the solution for the given equation.

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#### Worksheet 3

Solve the following equations.

1.	x + 9 = 15	2.	y - 6 = 9
3.	8 <i>x</i> = 24	4.	$\frac{x}{5} = 4$
5.	6 – <i>y</i> = 9	6.	x + 4 = 1
7.	$\frac{p}{3} = -8$	8.	<i>y</i> – 9 = 3
9.	17 <i>x</i> = - 85	10.	12 <i>x</i> = 12

#### Let us solve more equations.

**Example 6:** Solve 3x + 5 = 143x + 5 - 5 = 14 - 5Solution: — Add – 5 (the additive inverse of 5) to both sides 3x = 93x = 93 3 Divide both sides by 3 Therefore, x = 3 is the solution for the given equation. **Example 7:** Solve 3(x-2) = 123(x - 2)12 Solution: 3 3 ↓ Divide both sides by 3 x - 2 = 4x - 2 + 2 = 4 + 2Add 2 to both sides x = 6Therefore, x = 6 is the solution to the given equation. **Example 8:** Solve  $\frac{2x+5}{3} = 7$  $\frac{3 \times (2x+5)}{3} = 7 \times 3$  Multiply LHS and RHS by 3 Solution: 2x + 5 = 212x + 5 - 5 = 21 - 5 Subtracting 5 from LHS and RHS 2x = 16 $\frac{2x}{2} = \frac{16}{2}$  Divide LHS and RHS by 2 x = 8

Therefore, x = 8 is the solution to the given equation.

#### Worksheet 4

Solve the following equations.

1. 2x + 1 = 52. 6y - 5 = 19

- 3. 3 + 4y = -5 4.  $\frac{2}{3}p = 6$
- 5. 5y + 10 = 4y 10 6.  $\frac{1}{2}x + 3 = 5$
- 7. 3 (x + 1) = 6 8. 4 (x 2) = -8
- 9. 3x + 8 = 5x + 2 10  $\frac{1}{3}x + 11 = 14$

11. 
$$\frac{x-5}{4} = 3$$
 12.  $\frac{z+3}{4} = 17$ 

Let us now solve and check some equations.

**Example 9:** Solve 4x - 4 = 5 + x and check the solution.

**Solution:** 4x - 4 = 5 + x



Therefore, x = 3 is the solution for the equation 4x - 4 = 5 + x

**Check:** We check the solution by substituting the value x = 3 in the given equation.

LHS = 
$$4x - 4$$
  
=  $4 \times 3 - 4$   
=  $12 - 4$   
Substituting  $x = 3$   
=  $8$   
RHS =  $5 + x$   
=  $5 + 3$   
=  $8$   
Substituting  $x = 3$ 

We observe LHS = RHS for the value x = 3

**Example 10:** Solve  $\frac{2x}{3} = 16$  and check the solution. **Solution:**  $\frac{2x}{3} \times \frac{3}{2} = 16 \times \frac{3}{2}$  (Multiplying both sides by  $\frac{3}{2}$ ) Therefore, x = 24 Check: We substitute the value x = 24 in the equation.

LHS = 
$$\frac{2x}{3} = \frac{2 \times 24}{\frac{1}{3}}$$
  
= 16  
RHS = 16  
We have LHS = RHS for the value of  $x = 16$ 

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#### Worksheet 5

Solve and check the solution in the following equations.

- x + 7 = 91.
- $\frac{x}{4} = 25$ 2.
- 3. 9y = -135
- 4. 15 x = 4
- 5. 3(x-3) = 15
- 6. 7y + 3 = 9

# ALUE BASED QUESTIONS

- In a school out of 250 students of Class-VI, x students visited an old age home. 1. They entertained them with dance and music, they discussed about their families and children, helped them in cleaning their cupboards. But 75 children refused to visit.
  - (a) Find the number of students who visited the old age home.
  - (b) What value is shown by those students?
- 2. During an earthquake, x students from Class-VI donated ₹ 25 each and 25 students from Class-V donated ₹ 50 each. If total amount collected is ₹ 2000.
  - (a) Find the number of students of Class-VI who donated.
  - (b) What value is depicted by students?

## **BRAIN TEASERS**

1. A. Tick ( $\checkmark$ ) the correct answer. (a) Twelve less than four times a number is 20, is represented by-(i) 12 - 4x = 20(ii) 20 = -12 - 4x(iii) 4x + 12 = 20(iv) 4x - 12 = 20(b) Which of the following equations has x = 3 as a solution? (i) x - 2 = 5(ii) x + 2 = 5(iii) 2x + 1 = 0(iv) x + 4 = 6(c) The linear equation for 2 more than the sum of a and 4 is 8 will be-(i) 2a + 4 = 8(ii) 2(a + 4) = 8(iii) (a + 4) + 2 = 8(iv) a + 4 = 8(d) Which of the following is an equation of the given sentence? Anjali is 5 years older than Nanjani. (iii) N - 5 = A(i) 5 + A = N(ii) A + N = 5(iv) A = N + 5(e) Which of the following equations does not have a solution in integers? (i) x + 1 = 1(ii) x - 1 = 3(iii) 2x + 1 = 6 (iv) 1 - x = 5B. Answer the following questions. (a) If x + 5 = 7, then 2x - 3 = ?(b) What is the solution of the equation 5x + 5 = -40? (c) If 0.5y = 1.5, find y. (d) Check if p = 4 is the solution of  $\frac{1}{2}(p + 3) = 7$ (e) If  $\frac{x+2.4}{2} = 3$ , then find *x*. 2. Write any five linear equations. Which side of the following equations is binomial? 3.

(a) 
$$2x + 1 = 7$$

(b) 
$$\frac{4}{5}y + 2 = 4$$

(c) 2(x+3) = 8

#### 4. Convert into linear equations.

(a) Thrice a number decreased by 5 is 9.

- (b) A number multiplied by two is 3 more than the number itself.
- (c) Twice a number subtracted from 13 is 5.
- (d) One-ninth of a number added to one is 11.
- 5. Solve the following equations.
  - (a) 4x 3 = 2x + 1 (b) 2(x 5) = 10

(c) 
$$\frac{x+8}{2} = 4$$
 (d)  $0.5x = 25$ 

6. Solve the following equations and check the solutions.

(a) 
$$1 - 4x = -11$$
 (b)  $y + \frac{3}{2} = 5$ 

- 7. Complete the following statements with correct terms or signs.
  - (a) To solve the equation x + 3 = 8, we first add \_\_\_\_\_\_ to both sides.
  - (b) To solve the equation 9y = 72, we \_\_\_\_\_ LHS and RHS by 9.
  - (c) The solution of  $\frac{z}{3} = 5$  is \_\_\_\_\_.
  - (d) To solve the equation 2x 3 = 6, we first add \_\_\_\_\_\_ to both sides and then divide both sides by \_\_\_\_\_\_.
  - (e) The power of variable in a linear equation is \_\_\_\_\_.

## YOU MUST KNOW

- 1. Equation is a statement of equality which contains literal numbers (variables).
- 2. Linear equation means equation in which the power of the literal numbers (variables) is one.
- 3. Solving an equation means finding the value of the unknown (literal) in an equation.
- 4. While solving an equation same number is added on both sides of an equation, same number is subtracted on both sides of an equation. Similarly, we multiply or divide both sides by the same non-zero number.